

Noncommutativity relations in type IIB theory and their supersymmetry *

B. Nikolić [†] and B. Sazdović [‡]

Institute of Physics

University of Belgrade

P.O.Box 57, 11001 Belgrade, Serbia

May 10, 2010

Abstract

In the present paper we investigate noncommutativity of $D9$ and $D5$ -brane world-volumes embedded in space-time of type IIB superstring theory. Boundary conditions, which preserve half of the initial supersymmetry, are treated as canonical constraints. Solving the constraints we obtain original coordinates in terms of the effective coordinates and momenta. Presence of momenta induces noncommutativity of string endpoints. We show that noncommutativity relations are connected by $N = 1$ supersymmetry transformations and noncommutativity parameters are components of $N = 1$ supermultiplet.

PACS number(s): 02.40.Gh, 11.30.Pb, 11.25.Uv, 11.25.-w.

1 Introduction

In the present paper we investigate the noncommutativity of type IIB superstring theory [1] in pure spinor formulation (up to the quadratic terms) [2] using canonical approach. We consider two cases: when $D9$ -brane is space-time filling and when $D5$ -brane is embedded in space-time. Also we investigate the supersymmetry of noncommutativity relations.

The field content of R-R sector determines stable Dp -branes [1] in the certain superstring theory. The R-R sector of type IIB theory contains gauge fields $A_{(0)}$, $A_{(2)}$ and

*Work supported in part by the Serbian Ministry of Science and Technological Development, under contract No. 141036.

[†]e-mail address: bnikolic@ipb.ac.rs

[‡]e-mail address: sazdovic@ipb.ac.rs

$A_{(4)}$, and consequently, Dp -branes with odd value of p are stable. As a particular choice, besides $D9$ -brane, we will embed $D5$ -brane in ten dimensional space-time.

Spinorial part of ten dimensional superspace is spanned by two fermionic coordinates, θ^α and $\bar{\theta}^\alpha$ ($\alpha = 1, 2, \dots, 16$), which are Majorana-Weyl spinors. It is useful to express ten dimensional Majorana-Weyl spinor S^α in terms of two independent $D5$ -brane opposite chirality Weyl spinors, S^{α_1} and S^{α_2} ($\alpha_1, \alpha_2 = 1, 2, \dots, 8$) [1, 3, 4].

In the case of $D9$ -brane, when it fills all space-time, we chose Neumann boundary conditions for all bosonic coordinates x^μ . The boundary condition for fermionic coordinates, $(\theta^\alpha - \bar{\theta}^\alpha)|_0^\pi = 0$ produces additional one for their canonically conjugated momenta, $(\pi_\alpha - \bar{\pi}_\alpha)|_0^\pi = 0$. Choosing Neumann boundary conditions for x^i coordinates ($i = 0, 1, \dots, 5$), and Dirichlet boundary conditions for orthogonal ones x^a ($a = 6, \dots, 9$) we embed $D5$ -brane in ten dimensional space-time. For fermionic coordinates we choose boundary condition $[\theta^\alpha + (*\Gamma\bar{\theta})^\alpha]|_0^\pi = 0$, where $*\Gamma = \Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5$ is introduced to preserve supersymmetry [1]. Corresponding boundary condition for momenta is of the form $[\pi_\alpha + (*\Gamma\bar{\pi})_\alpha]|_0^\pi = 0$. In terms of $D5$ -brane spinors boundary conditions have the form $(\theta^{\alpha_1} - \bar{\theta}^{\alpha_1})|_0^\pi = 0$, $(\theta^{\alpha_2} + \bar{\theta}^{\alpha_2})|_0^\pi = 0$, $(\pi_{\alpha_1} - \bar{\pi}_{\alpha_1})|_0^\pi = 0$ and $(\pi_{\alpha_2} + \bar{\pi}_{\alpha_2})|_0^\pi = 0$.

We treat boundary conditions as canonical constraints [5, 6, 7, 8]. Using their consistency conditions, we rewrite them in compact σ -dependent form and find their Poisson brackets. It turns out that all constraints are of the second class for nonsingular open string metric $G^{eff} = G - 4BG^{-1}B$. Solving the second class constraints, we obtain initial coordinates in terms of effective coordinates and momenta. Presence of the momenta in the solutions for initial coordinates is source of noncommutativity. Noncommutativity relations are consistent with $N = 1$ supersymmetry transformations. We obtained that noncommutativity parameters contain only odd powers of background fields anti-symmetric under world-sheet parity transformation $\Omega : \sigma \rightarrow -\sigma$. They are components of $N = 1$ supermultiplet. This result represents a supersymmetric generalization of the result obtained by Seiberg and Witten [9].

At the end we give some concluding remarks.

2 Type IIB superstring theory and embedded $D5$ -brane

We will investigate pure spinor formulation [2, 10, 7, 8] of type IIB theory, neglecting ghost terms and keeping quadratic ones as in the action of Ref.[10].

The action in a flat background

$$S_0 = \int_{\Sigma} d^2\xi \left(\frac{\kappa}{2} \eta^{mn} \eta_{\mu\nu} \partial_m x^\mu \partial_n x^\nu - \pi_\alpha \partial_- \theta^\alpha + \partial_+ \bar{\theta}^\alpha \bar{\pi}_\alpha \right), \quad (2.1)$$

deformed by integrated form of the massless IIB supergravity vertex operator

$$V_{SG} = \int_{\Sigma} d^2\xi X_M^T A_{MN} \bar{X}_N, \quad (2.2)$$

produces the full action

$$S = S_0 + V_{SG}. \quad (2.3)$$

The world sheet (Σ) parameters are $\xi^m = (\tau, \sigma)$, while $D = 10$ -dimensional space-time coordinates are labelled by x^μ ($\mu = 0, 1, 2, \dots, 9$). The fermionic extension of space-time is expressed by same chirality fermionic coordinates θ^α and $\bar{\theta}^\alpha$. The variables π_α and $\bar{\pi}_\alpha$ are canonically conjugated to the coordinates θ^α and $\bar{\theta}^\alpha$, respectively. The fermionic coordinates and momenta are Majorana-Weyl spinors.

Using equations of motion which are consequences of BRST invariance, requiring for all background fields to be constant and restricted the action to the quadratic terms, the vertex operator gets the form

$$V_{SG} = \int_{\Sigma} d^2\xi \left[\kappa \left(\frac{1}{2} g_{\mu\nu} + B_{\mu\nu} \right) \partial_+ x^\mu \partial_- x^\nu - \pi_\alpha \Psi_\mu^\alpha \partial_- x^\mu + \partial_+ x^\mu \bar{\Psi}_\mu^\alpha \bar{\pi}_\alpha + \frac{1}{2\kappa} \pi_\alpha F^{\alpha\beta} \bar{\pi}_\beta \right], \quad (2.4)$$

where $g_{\mu\nu}$ is symmetric, $B_{\mu\nu}$ is antisymmetric Neveu-Schwarz field, Ψ_μ^α and $\bar{\Psi}_\mu^\alpha$ are NS-R gravitino fields and $F^{\alpha\beta}$ is R-R field strength. Adding V_{SG} to flat background action, we have

$$S = \kappa \int_{\Sigma} d^2\xi \left[\frac{1}{2} \eta^{mn} G_{\mu\nu} + \varepsilon^{mn} B_{\mu\nu} \right] \partial_m x^\mu \partial_n x^\nu + \int_{\Sigma} d^2\xi \left[-\pi_\alpha \partial_- (\theta^\alpha + \Psi_\mu^\alpha x^\mu) + \partial_+ (\bar{\theta}^\alpha + \bar{\Psi}_\mu^\alpha x^\mu) \bar{\pi}_\alpha + \frac{1}{2\kappa} \pi_\alpha F^{\alpha\beta} \bar{\pi}_\beta \right], \quad (2.5)$$

where $G_{\mu\nu} = \eta_{\mu\nu} + g_{\mu\nu}$ is constant gravitational field.

Embedding $D9$ -brane means that we choose Neumann boundary conditions for all space-time coordinates x^μ so that $D9$ -brane fills whole space-time. In order to embed $D5$ -brane in ten dimensional space-time we choose Neumann boundary conditions for x^i ($i = 0, 1, \dots, 5$) and Dirichlet boundary conditions for orthogonal directions x^a ($a = 6, 7, 8, 9$). The choice of background fields is the same as in Ref.[8] and the action is of the form

$$\begin{aligned} S &= \kappa \int_{\Sigma} d^2\xi \left[\frac{1}{2} \eta^{mn} G_{ij} + \varepsilon^{mn} B_{ij} \right] \partial_m x^i \partial_n x^j \\ &+ 2\Re \left\{ \int_{\Sigma} d^2\xi \left[-\pi_{\alpha_1} (\partial_\tau - \partial_\sigma) (\theta^{\alpha_1} + \Psi_i^{\alpha_1} x^i) + (\partial_\tau + \partial_\sigma) (\bar{\theta}^{\alpha_1} + \bar{\Psi}_i^{\alpha_1} x^i) \bar{\pi}_{\alpha_1} \right] \right\} \\ &+ 2\Re \left\{ \int_{\Sigma} d^2\xi \left[-\pi_{\alpha_2} (\partial_\tau - \partial_\sigma) (\theta^{\alpha_2} + \Psi_i^{\alpha_2} x^i) + (\partial_\tau + \partial_\sigma) (\bar{\theta}^{\alpha_2} + \bar{\Psi}_i^{\alpha_2} x^i) \bar{\pi}_{\alpha_2} \right] \right\} \\ &+ \frac{1}{\kappa} \Re \left\{ \int_{\Sigma} d^2\xi \left[\pi_{\alpha_1} f_{11}^{\alpha_1\beta_1} \bar{\pi}_{\beta_1} + \pi_{\alpha_1} f_{14}^{\alpha_1\beta_1} \bar{\pi}_{\beta_1}^* - \pi_{\alpha_2} f_{22}^{\alpha_2\beta_2} \bar{\pi}_{\beta_2} - \pi_{\alpha_2} f_{23}^{\alpha_2\beta_2} \bar{\pi}_{\beta_2}^* \right] \right\} \\ &+ \frac{1}{\kappa} \Re \left\{ \int_{\Sigma} d^2\xi \left[\pi_{\alpha_2} f_{21}^{\alpha_2\beta_1} \bar{\pi}_{\beta_1} - \pi_{\alpha_1} f_{12}^{\alpha_1\beta_2} \bar{\pi}_{\beta_2} + \pi_{\alpha_2} f_{24}^{\alpha_2\beta_1} \bar{\pi}_{\beta_1}^* - \pi_{\alpha_1} f_{13}^{\alpha_1\beta_2} \bar{\pi}_{\beta_2}^* \right] \right\}, \quad (2.6) \end{aligned}$$

where \Re means real part of some complex number, $*$ means complex conjugation and with f_{rs} we denoted 8 independent $D5$ -brane components of $F^{\alpha\beta}$ (for more details see Appendix B of Ref.[8]).

3 Canonical analysis

Here we will perform canonical analysis of type IIB superstring theory. Boundary conditions will be treated as canonical constraints. Consistency procedure for boundary conditions enable us to rewrite them in compact σ -dependent form. It turns out that all constraints are of the second class.

3.1 Hamiltonian

Using standard canonical procedure we find canonical Hamiltonian of type IIB superstring theory in the form

$$H_c = \int d\sigma \mathcal{H}_c, \quad \mathcal{H}_c = T_- - T_+, \quad T_{\pm} = t_{\pm} - \tau_{\pm}, \quad (3.1)$$

where

$$\begin{aligned} t_{\pm} &= \mp \frac{1}{4\kappa} G^{\mu\nu} I_{\pm\mu} I_{\pm\nu}, \quad I_{\pm\mu} = \pi_{\mu} + 2\kappa \Pi_{\pm\mu\nu} x'^{\nu} + \pi_{\alpha} \Psi_{\mu}^{\alpha} - \bar{\Psi}_{\mu}^{\alpha} \bar{\pi}_{\alpha}, \\ \tau_{+} &= (\theta'^{\alpha} + \Psi_{\mu}^{\alpha} x'^{\mu}) \pi_{\alpha} - \frac{1}{4\kappa} \pi_{\alpha} F^{\alpha\beta} \bar{\pi}_{\beta}, \quad \tau_{-} = (\bar{\theta}'^{\alpha} + \bar{\Psi}_{\mu}^{\alpha} x'^{\mu}) \bar{\pi}_{\alpha} + \frac{1}{4\kappa} \pi_{\alpha} F^{\alpha\beta} \bar{\pi}_{\beta}. \end{aligned} \quad (3.2)$$

For the case of embedded $D5$ -brane canonical Hamiltonian gets the form

$$\begin{aligned} t_{\pm} &= \mp \frac{1}{4\kappa} G^{ij} I_{\pm i} I_{\pm j}, \\ I_{\pm i} &= \pi_i + 2\kappa \Pi_{\pm ij} x'^j + 2\Re \left(\pi_{\alpha_1} \Psi_i^{\alpha_1} + \pi_{\alpha_2} \Psi_i^{\alpha_2} - \bar{\Psi}_i^{\alpha_1} \bar{\pi}_{\alpha_1} - \bar{\Psi}_i^{\alpha_2} \bar{\pi}_{\alpha_2} \right), \\ \tau_{+} &= 2\Re \left[(\theta'^{\alpha_1} + \Psi_i^{\alpha_1} x'^i) \pi_{\alpha_1} + (\theta'^{\alpha_2} + \Psi_i^{\alpha_2} x'^i) \pi_{\alpha_2} \right] \\ &\quad - \frac{1}{2\kappa} \Re \left(\pi_{\alpha_1} f_{11}^{\alpha_1\beta_1} \bar{\pi}_{\beta_1} + \pi_{\alpha_1} f_{14}^{\alpha_1\beta_1} \bar{\pi}_{\beta_1}^* - \pi_{\alpha_2} f_{22}^{\alpha_2\beta_2} \bar{\pi}_{\beta_2} - \pi_{\alpha_2} f_{23}^{\alpha_2\beta_2} \bar{\pi}_{\beta_2}^* \right) \\ &\quad - \frac{1}{2\kappa} \Re \left(\pi_{\alpha_2} f_{21}^{\alpha_2\beta_1} \bar{\pi}_{\beta_1} - \pi_{\alpha_1} f_{12}^{\alpha_1\beta_2} \bar{\pi}_{\beta_2} + \pi_{\alpha_2} f_{24}^{\alpha_2\beta_1} \bar{\pi}_{\beta_1}^* - \pi_{\alpha_1} f_{13}^{\alpha_1\beta_2} \bar{\pi}_{\beta_2}^* \right), \\ \tau_{-} &= 2\Re \left[(\bar{\theta}'^{\alpha_1} + \bar{\Psi}_i^{\alpha_1} x'^i) \bar{\pi}_{\alpha_1} + (\bar{\theta}'^{\alpha_2} + \bar{\Psi}_i^{\alpha_2} x'^i) \bar{\pi}_{\alpha_2} \right] \\ &\quad + \frac{1}{2\kappa} \Re \left(\pi_{\alpha_1} f_{11}^{\alpha_1\beta_1} \bar{\pi}_{\beta_1} + \pi_{\alpha_1} f_{14}^{\alpha_1\beta_1} \bar{\pi}_{\beta_1}^* - \pi_{\alpha_2} f_{22}^{\alpha_2\beta_2} \bar{\pi}_{\beta_2} - \pi_{\alpha_2} f_{23}^{\alpha_2\beta_2} \bar{\pi}_{\beta_2}^* \right) \\ &\quad + \frac{1}{2\kappa} \Re \left(\pi_{\alpha_2} f_{21}^{\alpha_2\beta_1} \bar{\pi}_{\beta_1} - \pi_{\alpha_1} f_{12}^{\alpha_1\beta_2} \bar{\pi}_{\beta_2} + \pi_{\alpha_2} f_{24}^{\alpha_2\beta_1} \bar{\pi}_{\beta_1}^* - \pi_{\alpha_1} f_{13}^{\alpha_1\beta_2} \bar{\pi}_{\beta_2}^* \right), \end{aligned} \quad (3.3)$$

and π_i , π_{α_1} , π_{α_2} , $\bar{\pi}_{\alpha_1}$ and $\bar{\pi}_{\alpha_2}$ are canonically conjugated to x^i , θ^{α_1} , θ^{α_2} , $\bar{\theta}^{\alpha_1}$ and $\bar{\theta}^{\alpha_2}$, respectively. Note, that in both cases energy-momentum tensor components T_{\pm} satisfy Virasoro algebra as a consequence of two dimensional diffeomorphisms.

3.2 Boundary conditions as canonical constraints

As a time translation generator Hamiltonian has to be differentiable with respect to coordinates and their canonically conjugated momenta. From this fact, following method

of Ref.[6], we will derive boundary conditions directly in terms of the canonical variables. Varying Hamiltonian H_c , we obtain

$$\delta H_c = \delta H_c^{(R)} - [\gamma_\mu^{(0)} \delta x^\mu + \pi_\alpha \delta \theta^\alpha + \delta \bar{\theta}^\alpha \bar{\pi}_\alpha] \Big|_0^\pi, \quad (3.4)$$

where $\delta H_c^{(R)}$ is regular term without τ and σ derivatives of supercoordinates and supermomenta variations and

$$\gamma_\mu^{(0)} = \Pi_{+\mu}{}^\nu I_{-\nu} + \Pi_{-\mu}{}^\nu I_{+\nu} + \pi_\alpha \Psi_\mu^\alpha + \bar{\Psi}_\mu^\alpha \bar{\pi}_\alpha. \quad (3.5)$$

Consequently, differentiability of Hamiltonian for type IIB theory demands

$$\left[\gamma_\mu^{(0)} \delta x^\mu + \pi_\alpha \delta \theta^\alpha + \delta \bar{\theta}^\alpha \bar{\pi}_\alpha \right] \Big|_0^\pi = 0. \quad (3.6)$$

Embedding $D9$ -brane implies Neumann boundary conditions for x^μ coordinates, which means

$$\gamma_\mu^{(0)} \Big|_0^\pi = 0. \quad (3.7)$$

Boundary condition for fermionic coordinates chosen to preserve half of the initial $N = 2$ supersymmetry is

$$(\theta^\alpha - \bar{\theta}^\alpha) \Big|_0^\pi = 0, \quad (3.8)$$

and it produces additional boundary condition for fermionic momenta

$$(\pi_\alpha - \bar{\pi}_\alpha) \Big|_0^\pi = 0. \quad (3.9)$$

In order to embed $D5$ -brane, for $D5$ -brane coordinates x^i we choose Neumann boundary conditions, implying

$$\begin{aligned} \gamma_i^{(0)} \Big|_0^\pi &= 0, \\ \gamma_i^{(0)} &= \Pi_{+i}{}^j I_{-j} + \Pi_{-i}{}^j I_{+j} + 2\Re \left(\pi_{\alpha_1} \Psi_i^{\alpha_1} + \pi_{\alpha_2} \Psi_i^{\alpha_2} + \bar{\Psi}_i^{\alpha_1} \bar{\pi}_{\alpha_1} + \bar{\Psi}_i^{\alpha_2} \bar{\pi}_{\alpha_2} \right). \end{aligned} \quad (3.10)$$

For othogonal coordinates we choose Dirichlet ones, $\delta x^a \Big|_0^\pi = 0$. As in Ref.[8], dynamics of x^a directions decouples from the rest part of action and we will not consider this boundary condition. Fermionic boundary conditions take the form

$$[\theta^\alpha + (*\Gamma\bar{\theta})^\alpha] \Big|_0^\pi = 0, \quad [\pi_\alpha + (*\Gamma\bar{\pi})_\alpha] \Big|_0^\pi = 0, \quad (3.11)$$

where $*\Gamma = \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^4 \Gamma^5$. By convention we introduce $*\Gamma$ because if Q_1 and Q_2 are type IIB supersymmetry charges then, after Dp -brane is embedded, the conserved supersymmetry is the linear combination [1]

$$Q_1 + \Gamma^0 \Gamma^1 \dots \Gamma^p Q_2. \quad (3.12)$$

Note that arbitrary Majorana-Weyl spinor can be expressed in terms of two opposite chirality $D5$ -brane Weyl spinors

$$S^\alpha = \begin{pmatrix} S^{\alpha_1} \\ S^{\alpha_2} \\ (b_1 S^*)^{\alpha_2} \\ -(b_1 S^*)^{\alpha_1} \end{pmatrix}, \quad (3.13)$$

where b_1 is $D5$ -brane complex conjugation operator. In terms of $D5$ -brane spinors boundary conditions takes the form

$$(\theta^{\alpha_1} - \bar{\theta}^{\alpha_1})|_0^\pi = 0, \quad (\theta^{\alpha_2} + \bar{\theta}^{\alpha_2})|_0^\pi = 0, \quad (3.14)$$

$$(\pi_{\alpha_1} - \bar{\pi}_{\alpha_1})|_0^\pi = 0, \quad (\pi_{\alpha_2} + \bar{\pi}_{\alpha_2})|_0^\pi = 0. \quad (3.15)$$

According with Ref.[6], we will treat the expressions (3.7)-(3.9) and (3.10), (3.14) and (3.15) as canonical constraints for $D9$ and $D5$ -brane, respectively.

3.3 Consistency of constraints

We assume that all background fields are constant which enables us to calculate Poisson brackets. Using standard Poisson algebra, the consistency procedure for $\gamma_\mu^{(0)}$ produces an infinite set of constraints

$$\gamma_\mu^{(n)} \equiv \{H_c, \gamma_\mu^{(n-1)}\} \quad (n = 1, 2, 3, \dots), \quad (3.16)$$

which can be rewritten at $\sigma = 0$ in the compact σ -dependent form

$$\Gamma_\mu(\sigma) \equiv \sum_{n=0}^{\infty} \frac{\sigma^n}{n!} \gamma_\mu^{(n)}|_0 = \Pi_{+\mu}{}^\nu I_{-\nu}(\sigma) + \Pi_{-\mu}{}^\nu I_{+\nu}(-\sigma) + \pi_\alpha(-\sigma) \Psi_\mu^\alpha + \bar{\Psi}_\mu^\alpha \bar{\pi}_\alpha(\sigma). \quad (3.17)$$

From conditions $(\theta^\alpha - \bar{\theta}^\alpha)|_0 = 0$ and $(\pi_\alpha - \bar{\pi}_\alpha)|_0 = 0$, we get

$$\Gamma^\alpha(\sigma) = \Theta^\alpha(\sigma) - \bar{\Theta}^\alpha(\sigma), \quad \Gamma_\alpha^\pi(\sigma) \equiv \Pi_\alpha(\sigma) - \bar{\Pi}_\alpha(\sigma), \quad (3.18)$$

where the right-hand side functions are defined as

$$\Theta^\alpha(\sigma) = \theta^\alpha(-\sigma) - \Psi_\mu^\alpha \tilde{q}^\mu(\sigma) - \frac{1}{2\kappa} F^{\alpha\beta} \int_0^\sigma d\sigma_1 P_s \bar{\pi}_\beta + \frac{1}{2\kappa} G^{\mu\nu} \Psi_\mu^\alpha \int_0^\sigma d\sigma_1 P_s (I_{+\nu} + I_{-\nu}), \quad (3.19)$$

$$\bar{\Theta}^\alpha(\sigma) = \bar{\theta}^\alpha(\sigma) + \bar{\Psi}_\mu^\alpha \tilde{q}^\mu(\sigma) + \frac{1}{2\kappa} F^{\beta\alpha} \int_0^\sigma d\sigma_1 P_s \pi_\beta + \frac{1}{2\kappa} G^{\mu\nu} \bar{\Psi}_\mu^\alpha \int_0^\sigma d\sigma_1 P_s (I_{+\nu} + I_{-\nu}), \quad (3.20)$$

$$\Pi_\alpha(\sigma) = \pi_\alpha(-\sigma), \quad \bar{\Pi}_{\bar{\alpha}}(\sigma) = \bar{\pi}_{\bar{\alpha}}(\sigma). \quad (3.21)$$

Similarly, for $D5$ -brane boundary conditions (3.10), (3.14) and (3.15) we get

$$\begin{aligned} \Gamma_i(\sigma) &= \Pi_{+i}^j I_{-j}(\sigma) + \Pi_{-i}^j I_{+j}(-\sigma) \\ &+ 2\Re \left[\pi_{\alpha_1}(-\sigma) \Psi_i^{\alpha_1} + \pi_{\alpha_2}(-\sigma) \Psi_i^{\alpha_2} + \bar{\Psi}_i^{\alpha_1} \bar{\pi}_{\alpha_1}(\sigma) + \bar{\Psi}_i^{\alpha_2} \bar{\pi}_{\alpha_2}(\sigma) \right], \end{aligned} \quad (3.22)$$

$$\Gamma^{\alpha_1}(\sigma) = \Theta^{\alpha_1}(\sigma) - \bar{\Theta}^{\alpha_1}(\sigma), \quad \Gamma^{\alpha_2}(\sigma) = \Theta^{\alpha_2}(\sigma) + \bar{\Theta}^{\alpha_2}(\sigma), \quad (3.23)$$

$$\Gamma_{\alpha_1}^\pi(\sigma) = \pi_{\alpha_1}(-\sigma) - \bar{\pi}_{\alpha_1}(\sigma), \quad \Gamma_{\alpha_2}^\pi(\sigma) = \pi_{\alpha_2}(-\sigma) + \bar{\pi}_{\alpha_2}(\sigma), \quad (3.24)$$

where right-hand side variables are defined as

$$\begin{aligned} \Theta^{\alpha_1}(\sigma) &= \theta^{\alpha_1}(-\sigma) - \Psi_i^{\alpha_1} \tilde{q}^i(\sigma) - \frac{1}{2\kappa} f_{11}^{\alpha_1 \beta_1} \int_0^\sigma d\sigma_1 P_s \bar{\pi}_{\beta_1} - \frac{1}{2\kappa} f_{14}^{\alpha_1 \beta_1} \int_0^\sigma d\sigma_1 P_s \bar{\pi}_{\beta_1}^* \\ &+ \frac{1}{2\kappa} f_{12}^{\alpha_1 \beta_2} \int_0^\sigma d\sigma_1 P_s \bar{\pi}_{\beta_2} + \frac{1}{2\kappa} f_{13}^{\alpha_1 \beta_2} \int_0^\sigma d\sigma_1 P_s \bar{\pi}_{\beta_2}^* + \frac{1}{2\kappa} G^{ij} \Psi_i^{\alpha_1} \int_0^\sigma d\sigma_1 P_s (I_{+j} + I_{-j}), \end{aligned} \quad (3.25)$$

$$\begin{aligned} \bar{\Theta}^{\alpha_1}(\sigma) &= \bar{\theta}^{\alpha_1}(\sigma) + \bar{\Psi}_i^{\alpha_1} \tilde{q}^i(\sigma) + \frac{1}{2\kappa} f_{11}^{\beta_1 \alpha_1} \int_0^\sigma d\sigma_1 P_s \pi_{\beta_1} + \frac{1}{2\kappa} f_{14}^{* \beta_1 \alpha_1} \int_0^\sigma d\sigma_1 P_s \pi_{\beta_1}^* \\ &+ \frac{1}{2\kappa} f_{21}^{\beta_2 \alpha_1} \int_0^\sigma d\sigma_1 P_s \pi_{\beta_2} + \frac{1}{2\kappa} f_{24}^{* \beta_2 \alpha_1} \int_0^\sigma d\sigma_1 P_s \pi_{\beta_2}^* + \frac{1}{2\kappa} G^{ij} \bar{\Psi}_i^{\alpha_1} \int_0^\sigma d\sigma_1 P_s (I_{+j} + I_{-j}). \end{aligned} \quad (3.26)$$

The expression for Θ^{α_2} can be obtained from the expression for Θ^{α_1} using substitution $\theta^{\alpha_1} \rightarrow \theta^{\alpha_2}$, $\pi_{\alpha_1} \leftrightarrow \pi_{\alpha_2}$, $\Psi_i^{\alpha_1} \rightarrow \Psi_i^{\alpha_2}$, $f_{11} \rightarrow -f_{22}$, $f_{14} \rightarrow -f_{23}$, $f_{12} \rightarrow -f_{21}$ and $f_{13} \leftrightarrow -f_{24}$. We obtain the expression for $\bar{\Theta}^{\alpha_2}$ from $\bar{\Theta}^{\alpha_1}$ using similar transition rules (fermionic variables and background fields have bars).

We introduced variables, even and odd under world-sheet parity transformation Ω : $\sigma \rightarrow -\sigma$. For bosonic variables we use standard notation [6]

$$q^\mu(\sigma) = P_s x^\mu(\sigma), \quad \tilde{q}^\mu(\sigma) = P_a x^\mu(\sigma), \quad (3.27)$$

$$p_\mu(\sigma) = P_s \pi_\mu(\sigma), \quad \tilde{p}_\mu(\sigma) = P_a \pi_\mu(\sigma), \quad (3.28)$$

while for fermionic ones we explicitly use the projectors on Ω even and odd parts

$$P_s = \frac{1}{2}(1 + \Omega), \quad P_a = \frac{1}{2}(1 - \Omega). \quad (3.29)$$

For all constraints we apply the consistency procedure at $\sigma = \pi$ and obtain similar expressions, where all variables depending on $-\sigma$ are replaced by the same variables depending on $2\pi - \sigma$. That set of constraints is solved by 2π periodicity of all canonical variables as in Ref.[6].

3.4 Classification of constraints

Let us denote all constraints with $\Gamma_A = (\Gamma_\mu, \Gamma^\alpha, \Gamma_\alpha^\pi)$. From

$$\{H_c, \Gamma_A\} = \Gamma'_A \approx 0, \quad (3.30)$$

it follows that all constraints weakly commute with canonical Hamiltonian, so there are no more constraints in the theory and the consistency procedure is completed.

For practical reasons we will separate the constraints Γ_A in two sets: the zero modes $(\theta^\alpha - \bar{\theta}^\alpha)|_0$ and the rest $^*\Gamma_A = (\Gamma_\mu, \Gamma'^\alpha, \Gamma_\alpha^\pi)$. The reason for this separation is that Poisson brackets of constraints $^*\Gamma_A$ close on δ' function while those with Γ_A , close on δ , δ' or step function.

First we will classify $^*\Gamma_A$. The algebra of the constraints $^*\Gamma_A$ has the form

$$\{^*\Gamma_A, ^*\Gamma_B\} = M_{AB}\delta', \quad (3.31)$$

where the supermatrix M_{AB} is given by the expression

$$M_{AB} = \begin{pmatrix} (M_1)_{\mu\nu} & (M_2)_\mu{}^\gamma{}_\delta \\ (M_3)^\alpha{}_{\beta\nu} & (M_4)^\alpha{}_{\beta}{}^\gamma{}_\delta \end{pmatrix} = \left(\begin{array}{c|cc} -\kappa G_{\mu\nu}^{eff} & -2(\Psi_{eff})_\mu^\gamma & 0 \\ -2(\Psi_{eff})_\nu^\alpha & \frac{1}{\kappa} F_{eff}^{\alpha\gamma} & -2\delta^\alpha{}_\delta \\ \hline 0 & -2\delta_\beta^\gamma & 0 \end{array} \right). \quad (3.32)$$

Here we introduced effective background fields

$$\begin{aligned} G_{\mu\nu}^{eff} &= G_{\mu\nu} - 4B_{\mu\rho}G^{\rho\lambda}B_{\lambda\nu}, & (\Psi_{eff})_\mu^\alpha &= \frac{1}{2}\Psi_{+\mu}^\alpha + B_{\mu\rho}G^{\rho\nu}\Psi_{-\nu}^\alpha, \\ F_{eff}^{\alpha\beta} &= F_a^{\alpha\beta} - \Psi_{-\mu}^\alpha G^{\mu\nu}\Psi_{-\nu}^\beta, \end{aligned} \quad (3.33)$$

and useful notation

$$\Psi_{\pm\mu}^\alpha = \Psi_\mu^\alpha \pm \bar{\Psi}_\mu^\alpha, \quad F_s^{\alpha\beta} = \frac{1}{2}(F^{\alpha\beta} + F^{\beta\alpha}), \quad F_a^{\alpha\beta} = \frac{1}{2}(F^{\alpha\beta} - F^{\beta\alpha}). \quad (3.34)$$

Following [9] we will refer to the fields appearing in matrix M_{AB} as the *open string* background fields. This is supersymmetric generalization of Seiberg and Witten open string metric, $G_{\mu\nu}^{eff}$, because all effective fields contain bilinear combinations of Ω odd fields.

When $D5$ -brane is embedded in ten dimensional space-time, the boundary conditions $^*\Gamma_A = (\Gamma_i, \Gamma'^{\alpha_1}, \Gamma'^{\alpha_2}, \Gamma_{\alpha_1}^\pi, \Gamma_{\alpha_2}^\pi)$ satisfy the algebra (3.31), where the supermatrix M_{AB} is given by the expression

$$M_{AB} = \left(\begin{array}{c|cccc} -\kappa G_{ij}^{eff} & -2(\Psi^{eff})_i^{\gamma_1} & -2(\Psi^{eff})_i^{\gamma_2} & 0 & 0 \\ -2(\Psi^{eff})^{\alpha_1}{}_j & \frac{1}{\kappa}(f_{11}^{eff})^{\alpha_1\gamma_1} & \frac{1}{\kappa}(f_{12}^{eff})^{\alpha_1\gamma_2} & -2\delta^{\alpha_1}{}_{\delta_1} & 0 \\ -2(\Psi^{eff})^{\alpha_2}{}_j & -\frac{1}{\kappa}(f_{12}^{eff})^{\alpha_2\gamma_1} & \frac{1}{\kappa}(f_{22}^{eff})^{\alpha_2\gamma_2} & 0 & -2\delta^{\alpha_2}{}_{\delta_2} \\ 0 & -2\delta_{\beta_1}^{\gamma_1} & 0 & 0 & 0 \\ 0 & 0 & -2\delta_{\beta_2}^{\gamma_2} & 0 & 0 \end{array} \right). \quad (3.35)$$

The open string background fields are defined as

$$\begin{aligned}
G_{ij}^{eff} &= G_{ij} - 4B_{ik}G^{kl}B_{lj}, \\
(\Psi_{eff})_i^{\alpha_1} &= \frac{1}{2}\Psi_{+i}^{\alpha_1} + B_{ik}G^{kj}\Psi_{-j}^{\alpha_1}, \quad (\Psi_{eff})_i^{\alpha_2} = \frac{1}{2}\Psi_{-i}^{\alpha_2} + B_{ik}G^{kj}\Psi_{+j}^{\alpha_2}, \\
(f_{11}^{eff})^{\alpha_1\beta_1} &= (f_{11}^a)^{\alpha_1\beta_1} - \Psi_{-i}^{\alpha_1}G^{ij}\Psi_{-j}^{\beta_1}, \quad (f_{22}^{eff})^{\alpha_2\beta_2} = (f_{22}^a)^{\alpha_2\beta_2} - \Psi_{+i}^{\alpha_2}G^{ij}\Psi_{+j}^{\beta_2}, \\
(f_{12}^{eff})^{\alpha_1\beta_2} &= \frac{1}{2}\left(f_{12}^{\alpha_1\beta_2} - f_{21}^{\beta_2\alpha_1}\right) - \Psi_{-i}^{\alpha_1}G^{ij}\Psi_{+j}^{\beta_2}.
\end{aligned} \tag{3.36}$$

From the definition of superdeterminant

$$s \det M_{AB} = \frac{\det(M_1 - M_2 M_4^{-1} M_3)}{\det M_4}, \tag{3.37}$$

and using the fact that

$$M_2 M_4^{-1} M_3 = 0, \quad \det M_4 = \text{const}, \tag{3.38}$$

we obtain from (3.32)

$$s \det M_{AB} \sim \det G^{eff}. \tag{3.39}$$

Because we assume that effective metric G^{eff} is nonsingular, we conclude that all constraints $^*\Gamma_A$ are of the second class. It is easy to check that zero modes, $(\theta^\alpha - \bar{\theta}^\alpha)|_0$, are also of the second class, and consequently, all constraints originating from boundary conditions, Γ_A , are of the second class. Note that the condition $s \det M_{AB} \neq 0$ is exactly the same condition as in the bosonic case [6].

4 Solution of the constraints

Instead to calculate Dirac brackets we prefer to explicitly solve second class constraints originating from boundary conditions. From $\Gamma_\mu = 0$, $\Gamma^\alpha = 0$ and $\Gamma_\alpha^\pi = 0$, we obtain

$$x^\mu(\sigma) = q^\mu - 2\Theta^{\mu\nu} \int_0^\sigma d\sigma_1 p_\nu + 2\Theta^{\mu\alpha} \int_0^\sigma d\sigma_1 p_\alpha, \quad \pi_\mu = p_\mu, \tag{4.1}$$

$$\theta^\alpha(\sigma) = \Phi^\alpha(\sigma) + \frac{1}{2}\tilde{\xi}^\alpha, \quad \pi_\alpha = p_\alpha + \tilde{p}_\alpha, \tag{4.2}$$

$$\bar{\theta}^\alpha(\sigma) = \Phi^\alpha(\sigma) - \frac{1}{2}\tilde{\xi}^\alpha, \quad \bar{\pi}_\alpha = p_\alpha - \tilde{p}_\alpha, \tag{4.3}$$

where

$$\Phi^\alpha(\sigma) = \frac{1}{2}\xi^\alpha - \Theta^{\mu\alpha} \int_0^\sigma d\sigma_1 p_\mu - \Theta^{\alpha\beta} \int_0^\sigma d\sigma_1 p_\beta, \tag{4.4}$$

$$\begin{aligned}\frac{1}{2}\xi^\alpha &\equiv P_s\theta^\alpha = P_s\bar{\theta}^\alpha, \quad \tilde{\xi}^\alpha \equiv P_a(\theta^\alpha - \bar{\theta}^\alpha), \\ p_\alpha &\equiv P_s\pi_\alpha = P_s\bar{\pi}_\alpha, \quad \tilde{p}_\alpha \equiv P_a\pi_\alpha = -P_a\bar{\pi}_\alpha,\end{aligned}\tag{4.5}$$

and

$$\Theta^{\mu\nu} = -\frac{1}{\kappa}(G_{eff}^{-1}BG^{-1})^{\mu\nu}, \quad \Theta^{\mu\alpha} = 2\Theta^{\mu\nu}(\Psi_{eff})_\nu^\alpha - \frac{1}{2\kappa}G^{\mu\nu}\psi_{-\nu}^\alpha, \tag{4.6}$$

$$\begin{aligned}\Theta^{\alpha\beta} &= \frac{1}{2\kappa}F_s^{\alpha\beta} + 4(\Psi_{eff})_\mu^\alpha\Theta^{\mu\nu}(\Psi_{eff})_\nu^\beta - \frac{1}{\kappa}\Psi_{-\mu}^\alpha(G^{-1}BG^{-1})^{\mu\nu}\Psi_{-\nu}^\beta \\ &+ \frac{G^{\mu\nu}}{\kappa}\left[\Psi_{-\mu}^\alpha(\Psi_{eff})_\nu^\beta + \Psi_{-\mu}^\beta(\Psi_{eff})_\nu^\alpha\right].\end{aligned}\tag{4.7}$$

Using σ -dependent form of boundary conditions (3.22)-(3.24), we get $D5$ -brane variables in terms of effective ones

$$x^i(\sigma) = q^i - 2\Theta^{ij}\int_0^\sigma d\sigma_1 p_j + 4\Re\left(\Theta^{i\alpha_1}\int_0^\sigma d\sigma_1 p_{\alpha_1} + \Theta^{i\alpha_2}\int_0^\sigma d\sigma_1 p_{\alpha_2}\right), \quad \pi_i = p_i, \tag{4.8}$$

$$\theta^{\alpha_1}(\sigma) = \Phi^{\alpha_1}(\sigma) + \frac{1}{2}\tilde{\xi}^{\alpha_1}(\sigma), \quad \pi_{\alpha_1} = p_{\alpha_1} + \tilde{p}_{\alpha_1}, \tag{4.9}$$

$$\theta^{\alpha_2}(\sigma) = \Phi^{\alpha_2}(\sigma) + \frac{1}{2}\tilde{\xi}^{\alpha_2}(\sigma), \quad \pi_{\alpha_2} = p_{\alpha_2} + \tilde{p}_{\alpha_2}, \tag{4.10}$$

$$\bar{\theta}^{\alpha_1}(\sigma) = \Phi^{\alpha_1}(\sigma) - \frac{1}{2}\tilde{\xi}^{\alpha_1}(\sigma), \quad \bar{\pi}_{\alpha_1} = p_{\alpha_1} - \tilde{p}_{\alpha_1}, \tag{4.11}$$

$$\bar{\theta}^{\alpha_2}(\sigma) = -\Phi^{\alpha_2}(\sigma) + \frac{1}{2}\tilde{\xi}^{\alpha_2}(\sigma), \quad \bar{\pi}_{\alpha_2} = -p_{\alpha_2} + \tilde{p}_{\alpha_2}, \tag{4.12}$$

where

$$\begin{aligned}\Phi^{\alpha_1}(\sigma) &= \frac{1}{2}\xi^{\alpha_1} - \Theta^{i\alpha_1}\int_0^\sigma d\sigma_1 p_i - \Theta^{\alpha_1\beta_1}\int_0^\sigma d\sigma_1 p_{\beta_1} - \Theta^{\alpha_1\beta_2}\int_0^\sigma d\sigma_1 p_{\beta_2} \\ &- {}^*\Theta^{\alpha_1\beta_1}\int_0^\sigma d\sigma_1 p_{\beta_1}^* - {}^*\Theta^{\alpha_1\beta_2}\int_0^\sigma d\sigma_1 p_{\beta_2}^*,\end{aligned}\tag{4.13}$$

$$\begin{aligned}\Phi^{\alpha_2}(\sigma) &= \frac{1}{2}\xi^{\alpha_2} - \Theta^{i\alpha_2}\int_0^\sigma d\sigma_1 p_i - \Theta^{\alpha_2\beta_1}\int_0^\sigma d\sigma_1 p_{\beta_1} - \Theta^{\alpha_2\beta_2}\int_0^\sigma d\sigma_1 p_{\beta_2} \\ &- {}^*\Theta^{\alpha_2\beta_1}\int_0^\sigma d\sigma_1 p_{\beta_1}^* - {}^*\Theta^{\alpha_2\beta_2}\int_0^\sigma d\sigma_1 p_{\beta_2}^*,\end{aligned}\tag{4.14}$$

and the coefficients multiplying momenta are of the form

$$\Theta^{ij} = -\frac{1}{\kappa}(G_{eff}^{-1}BG^{-1})^{ij}, \tag{4.15}$$

$$\Theta^{i\alpha_1} = 2\Theta^{ij}(\Psi_{eff})_j^{\alpha_1} - \frac{1}{2\kappa}G^{ij}\Psi_{-j}^{\alpha_1}, \quad \Theta^{i\alpha_2} = 2\Theta^{ij}(\Psi_{eff})_j^{\alpha_2} - \frac{1}{2\kappa}G^{ij}\Psi_{+j}^{\alpha_2}, \tag{4.16}$$

$$\begin{aligned}
\Theta^{\alpha_1\beta_1} &= \frac{1}{2\kappa}(f_{11}^s)^{\alpha_1\beta_1} + 4(\Psi_{eff})_i^{\alpha_1}\Theta^{ij}(\Psi_{eff})_j^{\beta_1} - \frac{1}{\kappa}\Psi_{-i}^{\alpha_1}(G^{-1}BG^{-1})^{ij}\Psi_{-j}^{\beta_1} \\
&+ \frac{G^{ij}}{\kappa} \left[\Psi_{-i}^{\alpha_1}(\Psi_{eff})_j^{\beta_1} + \Psi_{-i}^{\beta_1}(\Psi_{eff})_j^{\alpha_1} \right], \quad (4.17)
\end{aligned}$$

$$\begin{aligned}
\Theta^{\alpha_1\beta_2} &= \Theta^{\beta_2\alpha_1} = \frac{1}{4\kappa}(f_{12}^{\alpha_1\beta_2} + f_{21}^{\beta_2\alpha_1}) + 4(\Psi_{eff})_i^{\alpha_1}\Theta^{ij}(\Psi_{eff})_j^{\beta_2} \\
&- \frac{1}{\kappa}\Psi_{-i}^{\alpha_1}(G^{-1}BG^{-1})^{ij}\Psi_{+j}^{\beta_2} + \frac{G^{ij}}{\kappa} \left[\Psi_{-i}^{\alpha_1}(\Psi_{eff})_j^{\beta_2} + \Psi_{+i}^{\beta_2}(\Psi_{eff})_j^{\alpha_1} \right], \quad (4.18)
\end{aligned}$$

$$\begin{aligned}
\Theta^{\alpha_1\beta_1} &= \frac{1}{4\kappa}(f_{14}^{\alpha_1\beta_1} + f_{14}^{\beta_1\alpha_1}) + 4(\Psi_{eff})_i^{\alpha_1}\Theta^{ij}(\Psi_{eff})_j^{*\beta_1} - \frac{1}{\kappa}\Psi_{-i}^{\alpha_1}(G^{-1}BG^{-1})^{ij}\Psi_{-j}^{*\beta_1} \\
&+ \frac{G^{ij}}{\kappa} \left[\Psi_{-i}^{\alpha_1}(\Psi_{eff})_j^{*\beta_1} + \Psi_{-i}^{*\beta_1}(\Psi_{eff})_j^{\alpha_1} \right], \quad (4.19)
\end{aligned}$$

$$\begin{aligned}
*\Theta^{\alpha_1\beta_2} &= *\Theta^{\beta_2\alpha_1} = \frac{1}{4\kappa}(f_{13}^{\alpha_1\beta_2} + f_{24}^{*\beta_2\alpha_1}) + 4(\Psi_{eff})_i^{\alpha_1}\Theta^{ij}(\Psi_{eff})_j^{*\beta_2} \\
&- \frac{1}{\kappa}\Psi_{-i}^{\alpha_1}(G^{-1}BG^{-1})^{ij}\Psi_{+j}^{*\beta_2} + \frac{G^{ij}}{\kappa} \left[\Psi_{-i}^{\alpha_1}(\Psi_{eff})_j^{*\beta_2} + \Psi_{+i}^{*\beta_2}(\Psi_{eff})_j^{\alpha_1} \right]. \quad (4.20)
\end{aligned}$$

The coefficient $\Theta^{\alpha_2\beta_2}$ can be obtained from $\Theta^{\alpha_1\beta_1}$ after substitution $f_{11}^s \rightarrow f_{22}^s$, $(\Psi_{eff})_i^{\alpha_1} \rightarrow (\Psi_{eff})_i^{\alpha_2}$ and $\Psi_{-i}^{\alpha_1} \rightarrow \Psi_{+i}^{\alpha_2}$, while $*\Theta^{\alpha_2\beta_2}$ follows from $*\Theta^{\alpha_1\beta_1}$ after substitution $f_{14} \rightarrow f_{23}$, $(\Psi_{eff})_i^{\alpha_1} \rightarrow (\Psi_{eff})_i^{\alpha_2}$ and $\Psi_{-i}^{\alpha_1} \rightarrow \Psi_{+i}^{\alpha_2}$.

5 Noncommutativity of Dp -brane world-volume

Using the solutions of boundary conditions we will show that Poisson brackets of initial Dp -brane variables are nonzero.

5.1 $D9$ -brane

From basic Poisson algebra

$$\{x^\mu(\sigma), \pi_\nu(\bar{\sigma})\} = \delta^\mu{}_\nu \delta(\sigma - \bar{\sigma}), \quad (5.1)$$

and definitions (3.27)-(3.28) we obtain

$$\{q^\mu(\sigma), p_\nu(\bar{\sigma})\} = \delta^\mu{}_\nu \delta_s(\sigma, \bar{\sigma}), \quad \{\tilde{q}^\mu(\sigma), \tilde{p}_\nu(\bar{\sigma})\} = \delta^\mu{}_\nu \delta_a(\sigma, \bar{\sigma}), \quad (5.2)$$

where

$$\delta_s(\sigma, \bar{\sigma}) = \frac{1}{2} [\delta(\sigma - \bar{\sigma}) + \delta(\sigma + \bar{\sigma})], \quad \delta_a(\sigma, \bar{\sigma}) = \frac{1}{2} [\delta(\sigma - \bar{\sigma}) - \delta(\sigma + \bar{\sigma})], \quad (5.3)$$

are symmetric and antisymmetric delta functions, respectively. Using basic Poisson algebra of fermionic variables

$$\{\theta^\alpha(\sigma), \pi_\beta(\bar{\sigma})\} = \{\bar{\theta}^\alpha(\sigma), \bar{\pi}_\beta(\bar{\sigma})\} = -\delta^\alpha_\beta \delta(\sigma - \bar{\sigma}), \quad (5.4)$$

we have

$$\{P_s \theta^\alpha(\sigma) + P_s \bar{\theta}^\alpha(\sigma), P_s \pi_\beta(\bar{\sigma}) + P_s \bar{\pi}_\beta(\bar{\sigma})\} = -2\delta^\alpha_\beta \delta_s(\sigma, \bar{\sigma}), \quad (5.5)$$

which gives

$$\{\xi^\alpha(\sigma), p_\beta(\bar{\sigma})\} = -\delta^\alpha_\beta \delta_s(\sigma, \bar{\sigma}). \quad (5.6)$$

Similarly we obtain

$$\{\tilde{\xi}^\alpha(\sigma), \tilde{p}_\beta(\bar{\sigma})\} = -\delta^\alpha_\beta \delta_a(\sigma, \bar{\sigma}). \quad (5.7)$$

Therefore, the momenta p_μ , \tilde{p}_μ , p_α and \tilde{p}_α are canonically conjugated to the coordinates q^μ , \tilde{q}^μ , ξ^α and $\tilde{\xi}^\alpha$, respectively.

Using the solutions of constraints (4.1)-(4.3), we get the noncommutativity relations

$$\{x^\mu(\sigma), x^\nu(\bar{\sigma})\} = 2\Theta^{\mu\nu} \theta(\sigma + \bar{\sigma}), \quad (5.8)$$

$$\{x^\mu(\sigma), \theta^\alpha(\bar{\sigma})\} = -\Theta^{\mu\alpha} \theta(\sigma + \bar{\sigma}), \quad \{\theta^\alpha(\sigma), \bar{\theta}^\beta(\bar{\sigma})\} = \frac{1}{2} \Theta^{\alpha\beta} \theta(\sigma + \bar{\sigma}), \quad (5.9)$$

where

$$\theta(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1/2 & \text{if } 0 < x < 2\pi \\ 1 & \text{if } x = 2\pi \end{cases} \quad (5.10)$$

After introducing center of mass variables

$$A(\sigma) = A_{cm} + \mathcal{A}(\sigma), \quad A_{cm} = \frac{1}{\pi} \int_0^\pi d\sigma A(\sigma), \quad (5.11)$$

where $A(\sigma)$ is arbitrary variable, we obtain

$$\{x^\mu(\sigma), x^\nu(\bar{\sigma})\} = \Theta^{\mu\nu} \Delta(\sigma + \bar{\sigma}), \quad (5.12)$$

$$\{x^\mu(\sigma), \theta^\alpha(\bar{\sigma})\} = -\frac{1}{2} \Theta^{\mu\alpha} \Delta(\sigma + \bar{\sigma}), \quad \{\theta^\alpha(\sigma), \bar{\theta}^\beta(\bar{\sigma})\} = \frac{1}{4} \Theta^{\alpha\beta} \Delta(\sigma + \bar{\sigma}). \quad (5.13)$$

The function $\Delta(\sigma + \bar{\sigma})$ is nonzero only at string endpoints

$$\Delta(x) = 2\theta(x) - 1 = \begin{cases} -1 & \text{if } x = 0 \\ 0 & \text{if } 0 < x < 2\pi \\ 1 & \text{if } x = 2\pi \end{cases} \quad (5.14)$$

and we conclude that interior of the string is commutative, while string endpoints are noncommutative.

5.2 D5-brane

Applying the same procedure as in the case of $D9$ -brane, we get $D5$ -brane noncommutativity relations

$$\{x^i(\sigma), x^j(\bar{\sigma})\} = \Theta^{ij} \Delta(\sigma + \bar{\sigma}), \quad (5.15)$$

$$\{x^i(\sigma), \theta^{\alpha_1}(\bar{\sigma})\} = -\frac{1}{2} \Theta^{i\alpha_1} \Delta(\sigma + \bar{\sigma}) \quad , \quad \{x^i(\sigma), \theta^{\alpha_2}(\bar{\sigma})\} = -\frac{1}{2} \Theta^{i\alpha_2} \Delta(\sigma + \bar{\sigma}), \quad (5.16)$$

$$\{\theta^{\alpha_1}(\sigma), \bar{\theta}^{\beta_1}(\bar{\sigma})\} = \frac{1}{4} \Theta^{\alpha_1\beta_1} \Delta(\sigma + \bar{\sigma}), \quad \{\theta^{\alpha_2}(\sigma), \bar{\theta}^{\beta_2}(\bar{\sigma})\} = -\frac{1}{4} \Theta^{\alpha_2\beta_2} \Delta(\sigma + \bar{\sigma}), \quad (5.17)$$

$$\{\theta^{\alpha_1}(\sigma), \bar{\theta}^{\beta_2}(\bar{\sigma})\} = -\frac{1}{4} \Theta^{\alpha_1\beta_2} \Delta(\sigma + \bar{\sigma}). \quad (5.18)$$

The parameters multiplying complex conjugated momenta denoted by star are absent in noncommutativity relations, because the solutions for initial fermionic coordinates do not depend on complex conjugated effective coordinates.

On the solutions of the boundary conditions original string variables depend both on effective coordinates and effective momenta, and that is a source of noncommutativity. In the supersymmetric case the presence of Ω odd fields $B_{\mu\nu}$, $\Psi_{-\mu}^\alpha$ and $F_s^{\alpha\beta}$ leads to noncommutativity of the supercoordinates. Nontrivial $B_{\mu\nu}$ leads to nonzero of all noncommutative parameters, $\Theta^{\mu\nu}$, $\Theta^{\mu\alpha}$ and $\Theta^{\alpha\beta}$. If only $\Psi_{-\mu}^\alpha$ is nontrivial, we have $\Theta^{\mu\nu} = 0$, but $\Theta^{\mu\alpha}$ and $\Theta^{\alpha\beta}$ are nonzero. Finally, if only $F_s^{\alpha\beta}$ is nontrivial then $\Theta^{\mu\nu} = 0$ and $\Theta^{\mu\alpha} = 0$, and only $\Theta^{\alpha\beta}$ is nonzero. The last case corresponds to the noncommutativity relations used in [11], where bosonic variables are commutative. This discussion is the same for $D5$ -brane up to the following replacing

$$\begin{aligned} \Psi_{\pm\mu}^\alpha &= \Psi_\mu^\alpha \pm \bar{\Psi}_\mu^\alpha \rightarrow \Psi_{\pm\mu}^\alpha = \Psi_\mu^\alpha \mp (*\Gamma\bar{\Psi}_\mu)^\alpha, \\ F_s^{\alpha\beta} &= \frac{1}{2}(F^{\alpha\beta} + F^{\beta\alpha}) \rightarrow F_s^{\alpha\beta} = -\frac{1}{2} \left[(F^*\Gamma)^{\alpha\beta} + (F^*\Gamma)^{\beta\alpha} \right]. \end{aligned} \quad (5.19)$$

6 Supersymmetry of noncommutativity relations

Here we will explicitly show that noncommutativity relations of $D9$ -brane coordinates, (5.8) and (5.9), are connected by $N = 1$ supersymmetry transformations. Because of the relation between $D9$ and $D5$ -brane spinors [8], the similar relations hold for $D5$ -brane supersymmetry.

The action of initial theory (2.5) is invariant under global $N = 2$ supersymmetry with parameters ϵ and $\bar{\epsilon}$. The supersymmetry transformations of the variables x^μ , θ^α and $\bar{\theta}^\alpha$ [12] are

$$\delta x^\mu = \bar{\epsilon}^\alpha \Gamma_{\alpha\beta}^\mu \theta^\beta + \epsilon^\alpha \Gamma_{\alpha\beta}^\mu \bar{\theta}^\beta, \quad \delta \theta^\alpha = \epsilon^\alpha, \quad \delta \bar{\theta}^\alpha = \bar{\epsilon}^\alpha, \quad (6.1)$$

while the transformation rules of constant background fields are

$$\delta G_{\mu\nu} = \epsilon_+^\alpha \Gamma_{[\mu\alpha\beta} \Psi_{+\nu]}^\beta - \epsilon_-^\alpha \Gamma_{[\mu\alpha\beta} \Psi_{-\nu]}^\beta, \quad \delta B_{\mu\nu} = \epsilon_+^\alpha \Gamma_{[\mu\alpha\beta} \Psi_{-\nu]}^\beta - \epsilon_-^\alpha \Gamma_{[\mu\alpha\beta} \Psi_{+\nu]}^\beta, \quad (6.2)$$

$$\delta \Psi_{+\mu}^\alpha = -\frac{1}{16} \epsilon_-^\beta \Gamma_{\mu\beta\gamma} F_s^{\gamma\alpha} - \frac{1}{16} \epsilon_+^\beta \Gamma_{\mu\beta\gamma} F_a^{\gamma\alpha}, \quad \delta \Psi_{-\mu}^\alpha = \frac{1}{16} \epsilon_+^\beta \Gamma_{\mu\beta\gamma} F_s^{\gamma\alpha} + \frac{1}{16} \epsilon_-^\beta \Gamma_{\mu\beta\gamma} F_a^{\gamma\alpha}, \quad (6.3)$$

$$\delta A^{(0)} = 0, \quad \delta A_{\mu\nu}^{(2)} = -\epsilon_+^\alpha \Gamma_{[\mu\alpha\beta} \Psi_{+\nu]}^\beta - \epsilon_-^\alpha \Gamma_{[\mu\alpha\beta} \Psi_{-\nu]}^\beta + A^{(0)} \delta B_{\mu\nu}, \quad (6.4)$$

$$\delta A_{\mu\nu\rho\sigma}^{(4)} = 2\epsilon_+^\alpha \Gamma_{[\mu\nu\rho\alpha\beta} \Psi_{-\sigma]}^\beta + 2\epsilon_-^\alpha \Gamma_{[\mu\nu\rho\alpha\beta} \Psi_{+\sigma]}^\beta + 6A_{[\mu\nu}^{(2)} \delta B_{\rho\sigma]}. \quad (6.5)$$

Here we used notation

$$\epsilon_\pm^\alpha = \epsilon^\alpha \pm \bar{\epsilon}^\alpha = \text{const.}, \quad \Gamma_{\mu_1\mu_2\dots\mu_k} \equiv \Gamma_{[\mu_1}\Gamma_{\mu_2}\dots\Gamma_{\mu_k]}, \quad (6.6)$$

and $[\]$ in the subscripts of the fields mean antisymmetrization of space-time indices between brackets. The potentials $A^{(0)}$ and $A_{\mu\nu\rho\sigma}^{(4)}$ correspond to the symmetric part of $F^{\alpha\beta}$

$$F_s^{\alpha\beta} = \frac{1}{2}(F^{\alpha\beta} + F^{\beta\alpha}), \quad (6.7)$$

and $A_{\mu\nu}^{(2)}$ to antisymmetric one

$$F_a^{\alpha\beta} = \frac{1}{2}(F^{\alpha\beta} - F^{\beta\alpha}). \quad (6.8)$$

More about connection between two descriptions of R-R sector is given in Ref.[4] and Appendix B of Ref.[8].

From the solution of boundary conditions (4.2)-(4.3) and supersymmetry transformations (6.1), we have

$$\delta\theta^\alpha(\sigma) = \delta\Phi^\alpha(\sigma) + \frac{1}{2}\delta\tilde{\xi}^\alpha(\sigma) = \epsilon^\alpha, \quad (6.9)$$

$$\delta\bar{\theta}^\alpha(\sigma) = \delta\Phi^\alpha(\sigma) - \frac{1}{2}\delta\tilde{\xi}^\alpha(\sigma) = \bar{\epsilon}^\alpha, \quad (6.10)$$

which gives

$$\delta\Phi^\alpha(\sigma) = \frac{1}{2}\epsilon_+^\alpha, \quad \delta\tilde{\xi}^\alpha(\sigma) = \epsilon_-^\alpha. \quad (6.11)$$

From the boundary conditions (3.8), with the help of supersymmetry transformations (6.1), it holds

$$\epsilon_-^\alpha = 0. \quad (6.12)$$

The starting $N = 2$ supersymmetry transformations (6.1), on the solution of boundary conditions, reduces to $N = 1$ supersymmetry transformations

$$\delta x^\mu(\sigma) = \epsilon_+^\alpha \Gamma_{\alpha\beta}^\mu \Phi^\beta(\sigma) = \epsilon_+^\alpha \Gamma_{\alpha\beta}^\mu \theta^\beta(\sigma) - \frac{1}{2} \epsilon_+^\alpha \Gamma_{\alpha\beta}^\mu \tilde{\xi}^\beta(\sigma), \quad \delta\theta^\alpha = \delta\bar{\theta}^\alpha = \frac{1}{2}\epsilon_+^\alpha, \quad (6.13)$$

which gives

$$\delta q^\mu = \frac{1}{2} \epsilon_+^\alpha \Gamma_{\alpha\beta}^\mu \xi^\beta, \quad \delta \xi^\alpha = \epsilon_+^\alpha, \quad \delta \tilde{\xi}^\alpha = 0. \quad (6.14)$$

From Ref.[13] we read the supersymmetry transformations for the momenta

$$\delta p_\alpha = \frac{1}{2} \epsilon_+^\beta \Gamma_{\beta\alpha}^\mu p_\mu, \quad \delta p_\mu = 0. \quad (6.15)$$

The truncation from $N = 2$ to $N = 1$ supersymmetry we can realize omitting transformations for $G_{\mu\nu}$, $\Psi_{+\mu}$ and F_a [12]. The rest fields make $N = 1$ supermultiplet with transformation rules

$$\delta B_{\mu\nu} = \epsilon_+^\alpha \Gamma_{[\mu\alpha\beta} \Psi_{-\nu]}^\beta, \quad \delta \Psi_{-\mu}^\alpha = \frac{1}{16} \epsilon_+^\beta \Gamma_{\mu\beta\gamma} F_s^{\gamma\alpha}, \quad \delta F_s^{\alpha\beta} = 0. \quad (6.16)$$

Using $N = 1$ SUSY transformations (6.13)-(6.15), we can find the supersymmetric transformations of the coefficients $\Theta^{\mu\nu}$, $\Theta^{\mu\alpha}$ and $\Theta^{\alpha\beta}$ multiplying the momenta in the solutions of boundary conditions. From

$$\begin{aligned} \delta x^\mu(\sigma) &= \frac{1}{2} \epsilon_+^\alpha \Gamma_{\alpha\beta}^\mu \xi^\beta - 2\delta\Theta^{\mu\nu} \int_0^\sigma d\sigma_1 p_\nu - 2\Theta^{\mu\nu} \int_0^\sigma d\sigma_1 \delta p_\nu, \\ &+ 2\delta\Theta^{\mu\alpha} \int_0^\sigma d\sigma_1 p_\alpha + 2\Theta^{\mu\alpha} \int_0^\sigma d\sigma_1 \delta p_\alpha = \epsilon_+^\alpha \Gamma_{\alpha\beta}^\mu \theta(\sigma)^\beta - \frac{1}{2} \epsilon_+^\alpha \Gamma_{\alpha\beta}^\mu \tilde{\xi}(\sigma)^\beta, \\ \delta\theta^\alpha(\sigma) &= \frac{1}{2} \epsilon_+^\alpha - \delta\Theta^{\mu\alpha} \int_0^\sigma d\sigma_1 p_\mu - \Theta^{\mu\alpha} \int_0^\sigma d\sigma_1 \delta p_\mu - \delta\Theta^{\alpha\beta} \int_0^\sigma d\sigma_1 p_\beta - \Theta^{\alpha\beta} \int_0^\sigma d\sigma_1 \delta p_\beta = \frac{1}{2} \epsilon_+^\alpha, \end{aligned} \quad (6.17)$$

$$(6.18)$$

we obtain global $N = 1$ SUSY transformations of the background fields

$$\delta\Theta^{\mu\nu} = \epsilon_+^\alpha \Gamma_{\alpha\beta}^{[\mu} \Theta^{\nu]\beta}, \quad \delta\Theta^{\mu\alpha} = -\frac{1}{2} \epsilon_+^\beta \Gamma_{\beta\gamma}^\mu \Theta^{\gamma\alpha}, \quad \delta\Theta^{\alpha\beta} = 0. \quad (6.19)$$

Consequently, these fields are components of $N = 1$ supermultiplet. The coefficients, $\Theta^{\mu\nu}$, $\Theta^{\mu\alpha}$ and $\Theta^{\alpha\beta}$, are the background fields of the T-dual theory. This explains the fact that their SUSY transformations have the same form as the transformations of the corresponding dual partners $B_{\mu\nu}$, $\Psi_{-\mu}^\alpha$ and $F_s^{\alpha\beta}$ (6.16).

Using $N = 1$ supersymmetry transformations of SUSY coordinates (6.13) and background fields (6.19), we can easily prove that noncommutativity relations, (5.8) and (5.9), are connected by supersymmetry transformations. The SUSY transformation of (5.8)

$$\epsilon_+^\alpha \Gamma_{\alpha\beta}^{[\mu} \{x^\nu], \theta^\beta\} = -\epsilon_+^\alpha \Gamma_{\alpha\beta}^{[\mu} \Theta^{\nu]\beta} \theta(\sigma + \bar{\sigma}), \quad (6.20)$$

produces the first relation in (5.9). Similarly, SUSY transformation of the first relation in (5.9)

$$\epsilon_+^\beta \Gamma_{\beta\gamma}^\mu \{\theta^\gamma, \theta^\alpha\} = \frac{1}{2} \epsilon_+^\beta \Gamma_{\beta\gamma}^\mu \Theta^{\gamma\alpha} \theta(\sigma + \bar{\sigma}), \quad (6.21)$$

produces the second relation in (5.9).

7 Concluding remarks

In this paper we considered noncommutativity properties and related supersymmetry transformations of $D9$ and $D5$ -branes in type IIB superstring theory. We used the pure spinor formulation of the theory introduced in Refs.[2, 10].

Following [5, 8, 6] we treated all boundary conditions at string endpoints as canonical constraints and checked their consistency. For nonsingular G^{eff} all constraints are of the second class. We can solve them and obtain the initial coordinates x^μ , θ^α and $\bar{\theta}^\alpha$ in terms of effective ones, q^μ , ξ^α and $\tilde{\xi}^\alpha$ (momenta independent parts of the solutions for initial supercoordinates x^μ , θ^α and $\bar{\theta}^\alpha$) and momenta p_μ and p_α (canonically conjugated to q^μ and ξ^α).

The fact that original string variables depend both on effective coordinates and effective momenta is a source of noncommutativity (5.12)-(5.13). The solution for initial variables do not depend on momenta \tilde{p}_α (canonically conjugated to $\tilde{\xi}^\alpha$). So, Ω odd variables, denoted with tilde, do not contribute to noncommutativity relations. Absence of the fermionic coordinates in the solution for x^μ implies that Poisson bracket $\{x^\mu, x^\nu\}$ is the same as in pure bosonic case. Similar conclusions are valid for $D5$ -brane.

Noncommutativity obtained in the present paper represents a generalization of the results from Ref.[10]. In special case, when $\Psi^\alpha = \bar{\Psi}_\mu^\alpha$, the noncommutativity relations (5.12)-(5.13) correspond to the relations of Ref.[10].

The result of the present paper can be considered as a supersymmetric generalization of the result obtained for bosonic string [9]. Beside $B_{\mu\nu}$, its superpartners $\Psi_{-\mu}^\alpha$ and $F_s^{\alpha\beta}$ are also a source of noncommutativity. For noncommutativity of bosonic coordinates it is necessary to have nontrivial $B_{\mu\nu}$. Noncommutativity of bosonic and fermionic coordinates can be caused by both $B_{\mu\nu}$ and $\Psi_{-\mu}^\alpha$, while noncommutativity of two fermionic coordinates can be caused by all components of noncommutative supermultiplet, $B_{\mu\nu}$, $\Psi_{-\mu}^\alpha$ and $F_s^{\alpha\beta}$.

Note that fermionic boundary conditions split $N = 2$ supermultiplet (consisting of background fields $G_{\mu\nu}$, $B_{\mu\nu}$, $\Psi_{+\mu}^\alpha$, $\Psi_{-\mu}^\alpha$ and $F_a^{\alpha\beta}$) into two $N = 1$ supermultiplets. One, Ω even ($G_{\mu\nu}$, $\Psi_{+\mu}^\alpha$, $F_a^{\alpha\beta}$), represents background fields of type I theory, and the second one, Ω odd ($B_{\mu\nu}$, $\Psi_{-\mu}^\alpha$, $F_s^{\alpha\beta}$) is source of noncommutativity of supercoordinates (x^μ, θ^α) .

References

- [1] J. Polchinski, *String theory - Volume II*, Cambridge University Press, 1998; K. Becker, M. Becker and J. H. Schwarz, *String Theory and M-Theory - A Modern Introduction*, Cambridge University Press, 2007.
- [2] N. Berkovits and P. Howe, *Nucl. Phys.* **B635** (2002) 75; P. A. Grassi, G. Policastro and P. van Nieuwenhuizen, *JHEP* **11** (2002) 004; P. A. Grassi, G. Policastro and

- P. van Nieuwenhuizen, *Adv. Theor. Math. Phys.* **7** (2003) 499; P. A. Grassi, G. Policastro and P. van Nieuwenhuizen, *Phys. Lett.* **B553** (2003) 96.
- [3] M. J. Duff, Ramzi R. Khuri and J. X. Lu, *Phys. Rept.* **259** (1995) 213.
- [4] E. Kiritsis, *Introduction to Superstring Theory*, Leuven University Press, 1998, hep-th/9709062.
- [5] F. Ardalan, H. Arfaei, M. M. Sheikh-Jabbari, *Nucl. Phys.* **B576**, 578 (2000); C. S. Chu, P. M. Ho, *Nucl. Phys.* **B568**, 447 (2000); T. Lee, *Phys. Rev.* **D62** (2000) 024022.
- [6] B. Sazdović, *Eur. Phys. J* **C44** (2005) 599; B. Nikolić and B. Sazdović, *Phys. Rev.* **D74** (2006) 045024; B. Nikolić and B. Sazdović, *Phys. Rev.* **D75** (2007) 085011; B. Nikolić and B. Sazdović, *Adv. Theor. Math. Phys.* **14** (2010) 1.
- [7] B. Nikolić and B. Sazdović, *Phys. Lett.* **B666** (2008) 400.
- [8] B. Nikolić and B. Sazdović, *Nucl. Phys. B* **836** (2010) 100–126.
- [9] N. Seiberg and E. Witten, *JHEP* **09** (1999) 032.
- [10] J. de Boer, P. A. Grassi and P. van Nieuwenhuizen, *Phys. Lett.* **B574** (2003) 98.
- [11] M. Dimitrijević, V. Radovanović and J. Wess, *JHEP* **12** (2007) 059.
- [12] F. Riccioni, *Phys. Lett.* **B560** (2003) 223; N. Berkovits, ICTP Lectures on Covariant Quantization of the Superstring, hep-th/0209059.
- [13] P.A. Grassi, G. Policastro, M. Porrati and P. van Nieuwenhuizen, *JHEP* **10** (2002) 054.